

Latest Trend for Interference Cancellation and Rate Maximization of the Cognitive Networks

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Abstract

Cognitive networks are one of the recent trends in wireless communication. But still some drawbacks in these types of networks, main one is the interference caused by the cognitive user to the primary user or primary user to the cognitive user. Which will totally affect the performance of the entire system, to overcome from these kinds of problems we can use a technique called beam forming vector design at both sides. The beam forming vectors are designed such that the interference caused by the cognitive transmitter to the primary receiver and the interference caused by the primary transmitter to the cognitive receiver is completely nullified while maximizing the rate of both the primary and secondary links. The proposed algorithms also maximize the achievable rates of both links through uncoordinated beam forming. Beam forming exploits channel knowledge at the transmitter to maximize the signal-to-noise ratio (SNR) at the receiver by transmitting in the direction of the eigenvector corresponding to the largest Eigen value of the channel.

Keywords: Cognitive Network, Signal to Noise Ratio (SNR), Beam Forming.

1. Introduction

Communication is the activity of conveying meaningful information. Communication requires a sender, a message, and an intended recipient, although the receiver need not be present or aware of the sender's intent to communicate at the time of communication; thus communication can occur across vast distances in time and space. The communication process is complete once the receiver has understood the message of the sender.

In recent years, the words cognitive and smart have become buzzwords that are applied to many different

networking and communications systems. The opportunistic use of the wireless spectrum has been a hot research topic in the wireless communications community in recent years due to the intense competition for the use of spectrum at frequencies below 3 GHz.

Cognitive network has a cognitive process that can perceive current network conditions, and then plan, decide and act on those conditions. The network can learn from these adaptations and use them to make future decisions; all while taking into account end to end goals.

A cognitive network consists of a number of traditional wireless service subscribers and they are called as cognitive users. The traditional wireless service subscribers have the legacy priority access to the spectrum and are usually called primary users in this network. Cognitive users presented in this system are also known as the secondary users, are allowed to access the spectrum only if communication does not create significant interference to the licensed primary users.

The Cognitive Radio (CR) concept is a new wireless communication paradigm that improves the spectrum usage efficiency by exploiting the existence of spectrum holes.

CRNs are networks that have cognitive and reconfigurable properties and the capability to detect unoccupied spectrum holes and change frequency for end-to-end communication. In most of the existing proposals, CRNs employ three steps of basic functionality. Observing and sensing is the first step of the cognitive process. The next step is to identify and analyze the spectrum. The last step is sharing the spectrum information and executing spectrum assignment.

2. Beam Forming

Beamforming can be used for radio or sound waves. It has found numerous applications in radar, sonar, seismology, wireless communications, radio astronomy, acoustics, and biomedicine. Adaptive beamforming is used to detect and estimate the signal-of-interest at the output of a sensor array by means of optimal spatial filtering and interference rejection. Beam forming is a signal processing technique used in sensor arrays for directional signal transmission or reception. Beam forming can be used at both the transmitting and receiving ends in order to achieve spatial selectivity.

Beam forming techniques are mainly used to change the directionality of the array. When transmitting, a beamformer controls the phase and relative amplitude of the signal at each transmitter, in order to create a pattern of constructive and destructive interference in the wave front.

Beamforming techniques can be broadly divided into two categories

- i. conventional (fixed or switched beam) beamformers
- ii. Adaptive beamformers or phased array

Conventional beamformers use a fixed set of weightings and time-delays (or phasing's) to combine the signals from the sensors in the array, primarily using only information about the location of the sensors in space and the wave directions of interest. In contrast, adaptive beamforming techniques generally combine this information with properties of the signals actually received by the array, typically to improve rejection of unwanted signals from other directions.

All the weights of the antenna elements can have equal magnitudes. The beamformer is steered to a specified direction only by selecting appropriate phases for each antenna. If the noise is uncorrelated and there are no directional interferences, the signal-to-noise ratio of a beamformer is given by

$$SNR = \frac{1}{\sigma_N^2} \cdot P \quad (1)$$

Where P = Transmitting power, σ_N^2 = Noise Power

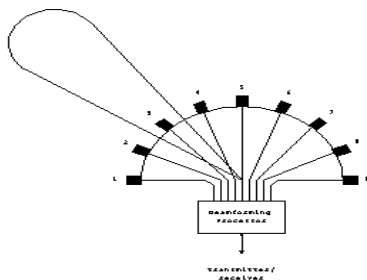


Fig.1 Beam Forming

2.1 Spectrum Sharing

According to conventional wisdom, we currently suffer from a shortage of spectrum. This supposedly limits our ability to introduce new wireless products and services such as ubiquitous broadband Internet access, limits our ability to make current systems like cellular telephony more common and less expensive, limits our ability to increase the data rates and ranges of existing products like wifi, and even limits our ability to provide firefighters, police, and paramedics with the communications systems they need to do their jobs.

In actuality, if one measures spectrum utilization (as CMU students have), it is clear that much of the spectrum sits idle at any given time. One reason is that we often prevent interference between systems by giving each system exclusive access to a block of spectrum. Thus, whenever such a system is not transmitting, spectrum sits idle. In this project, we seek new methods that allow disparate wireless systems to share spectrum without causing excessive harmful interference to their neighbors. Our goal is to increase the amount of communications that can take place in a given amount of spectrum by orders of magnitude, which would lead to a revolution in wireless products and services.

2.2 Orthogonal Frequency Division Multiple

Orthogonal frequency-division multiplexing (OFDM) is a method of encoding digital data on multiple carrier frequencies. OFDM has developed into a popular scheme for wideband digital communication, whether wireless or over copper wires, used in applications such as digital television and audio broadcasting, DSL broadband internet access, wireless networks, and 4G mobile communications.

Conceptually, OFDM is a specialized FDM, the additional constraint being: all the carrier signals are orthogonal to each other. In OFDM, the sub-carrier frequencies are chosen so that the sub-carriers are orthogonal to each other, meaning that cross-talk between the sub-channels is eliminated and inter-carrier guard bands are not required. This greatly simplifies the design of both the transmitter and the receiver; unlike conventional FDM, a separate filter for each sub-channel is not required. The orthogonality also allows high spectral efficiency, with a total symbol rate near the Nyquist rate for the equivalent baseband signal. Almost the whole available frequency band can be utilized.

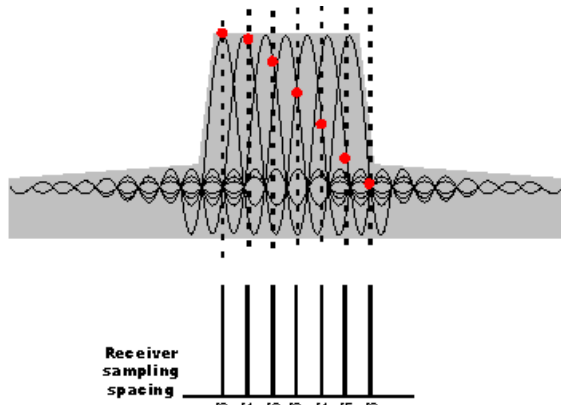


Fig.2 OFDM

OFDM requires very accurate frequency synchronization between the receiver and the transmitter; otherwise it produces crosstalk between the subcarrier signals.

2.3 Rayleigh and Rician Fading Channels

Rayleigh fading is a statistical model for the effect of a propagation environment on a radio signal, such as that used by wireless devices. Rayleigh fading models assume that the magnitude of a signal that has passed through such a transmission medium (also called a communications channel) will vary randomly, or fade, according to a Rayleigh distribution — the radial component of the sum of two uncorrelated Gaussian random variables. Rayleigh fading is viewed as a reasonable model for tropospheric and ionospheric signal propagation as well as the effect of heavily built-up urban environments on radio signals. Rayleigh fading is most applicable when there is no dominant propagation along a line of sight between the transmitter and receiver. If there is a dominant line of sight, Rician fading may be more applicable.

The requirement that there be many scatterers present means that Rayleigh fading can be a useful model in heavily built-up city centers where there is no line of sight between the transmitter and receiver and many buildings and other objects attenuate, reflect, refract, and diffract the signal. Experimental work in Manhattan has found near-Rayleigh fading there. In tropospheric and ionospheric signal propagation the many particles in the atmospheric layers act as scatterers and this kind of environment may also approximate Rayleigh fading. If the environment is such that, in addition to the scattering, there is a strongly dominant signal seen at the receiver, usually caused by a line of sight, then the mean of the random process will no longer be zero, varying instead around the

power-level of the dominant path. Such a situation may be better modelled as Rician fading.

Rician fading is a stochastic model for radio propagation anomaly caused by partial cancellation of a radio signal by itself — the signal arrives at the receiver by several different paths (hence exhibiting multipath interference), and at least one of the paths is changing (lengthening or shortening). Rician fading occurs when one of the paths, typically a line of sight signal, is much stronger than the others. In Rician fading, the amplitude gain is characterized by a Rician distribution. Rayleigh fading is the specialized model for stochastic fading when there is no line of sight signal, and is sometimes considered as a special case of the more generalized concept of Rician fading. In Rayleigh fading, the amplitude gain is characterized by a Rayleigh distribution.

3. Channel Model

3.1 Cognitive Network

Consider a cognitive network with a single primary user and a single cognitive (secondary) user as depicted in Fig. 3. Each user consists of a transmitter and a receiver. The primary transmitter and receiver are equipped with N_t^p and N_r^p antennas, respectively. Receiver is denoted by W whereas the one between the secondary transmitter and receiver is denoted by H. The interference channel from the primary transmitter to the secondary receiver is denoted by D and the interference channel from the secondary transmitter to the primary receiver is denoted by G.

We model the individual channel elements in W, H, D, and G. The primary transmitter employs a beam forming vector \mathbf{u} for the transmission of its data symbol x_p . At the cognitive link, the transmitter employs a beam forming vector \mathbf{f} for the transmission of its data symbol x_c . x_p and x_c are assumed to be complex zero-mean unit variance random variables. Furthermore, let \mathbf{v} and \mathbf{t} be the receiver combining vector for the primary and secondary receiver, respectively.

$$\{V_{opt}, f_{opt}, t_{opt}, u_{opt}\} = \text{argmax}\{\log_2(1 + SINR_p) + \log_2(1 + SINR_c)\}$$

$$\mathbf{v}, \mathbf{f}, \mathbf{t}, \mathbf{u}$$

$$\begin{cases} \mathbf{v}^* \mathbf{G} \mathbf{f} = 0 \text{ and } \mathbf{t}^* \mathbf{D} \mathbf{u} = 0 \\ \mathbf{u}^* \mathbf{u} = \mathbf{f}^* \mathbf{f} = \mathbf{v}^* \mathbf{v} = \mathbf{t}^* \mathbf{t} = 1 \end{cases}$$

$$\{V_{opt}, f_{opt}, t_{opt}, u_{opt}\} = \text{argmax}\{\log_2(1 + SINR_p) + \log_2(1 + SINR_c)\}$$

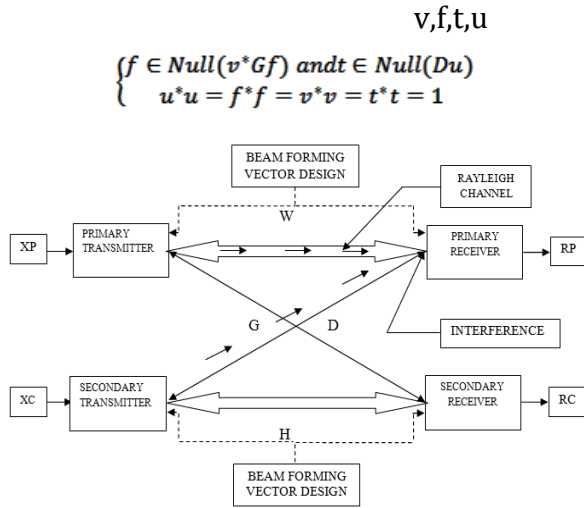


Fig.3. Proposed System

3.2 Beamforming Vector Design

In the cognitive network the secondary user (cognitive user) is transparent to the primary user since the performance of the primary user should not be affected by the secondary link. In these networks zero interference can be achieved by appropriately designing \mathbf{v} or \mathbf{f} and \mathbf{t} or \mathbf{u} . To achieve zero interference caused to the primary receiver, these condary transmitter can beamform in the null space of $\mathbf{V}^* \mathbf{G}$.

Likewise, at the cognitive receiver the receiver beamforming vector \mathbf{t} can be designed such that it is in the null space of $\mathbf{D}\mathbf{u}$ in order to avoid the interference caused by the primary transmitter. Note that $\mathbf{V}^* \mathbf{G}$ is a $1 \times N_c^e$ vector and the dimension of its null space is $N_c^e - 1$. Similarly, the dimension of $\mathbf{D}\mathbf{u}$ is $N_r^e \times 1$ and the dimension of its null space is $N_r^e - 1$. The rate of the primary user can be maximized by appropriately designing \mathbf{v} and \mathbf{u} . Since no interference is created at the primary user and the only constraint for the beamforming vectors \mathbf{v} and \mathbf{u} is the energy constraint.

The spectral efficiency can be maximized by maximizing the SINR due to the monotonic property of the logarithm function. It is well known that the SINR maximizing receive beamformer for a point-to-point link is the maximal ratio combining beamformer.

The basic beamforming vectors are given by

$$\begin{aligned} \{f_{opt}, t_{opt}\} = \underset{f, t}{\operatorname{argmax}} \left\{ \frac{P_c t^* H f^* H^* t}{t^* t} \right\} \\ \text{subject to } \begin{cases} f \in \text{Null}(\mathbf{V}_{opt}^* \mathbf{G}) \text{ and } t \in \text{Null}(\mathbf{D}\mathbf{u}_{opt}) \\ f^* f = t^* t = 1 \end{cases} \end{aligned} \quad (8)$$

The signal received at the primary receiver is rearranged according to the beamforming vectors, and given by

$$r_p = \frac{\sqrt{P_p} \mathbf{u}^* \mathbf{W}^* \mathbf{W} \mathbf{u}}{\sqrt{\mathbf{u}^* \mathbf{W}^* \mathbf{W} \mathbf{u}}} x_p + \frac{\mathbf{u}^* \mathbf{W}^*}{\sqrt{\mathbf{u}^* \mathbf{W}^* \mathbf{W} \mathbf{u}}} n_p \quad (9)$$

And the corresponding \mathbf{v}_{opt} is given by

$$\mathbf{v}_{opt} = \mathbf{W} \mathbf{u} / \sqrt{\mathbf{u}^* \mathbf{W}^* \mathbf{W} \mathbf{u}} \quad (10)$$

And the corresponding SINR is given by

$$\text{SINR}_p = \frac{P_p \mathbf{u}^* \mathbf{W}^* \mathbf{W} \mathbf{u}}{\sigma_p^2} \quad (11)$$

3.3 Discrete Search

Let F and T be the set of basis vectors which spans the null space of $\mathbf{V}_{opt}^* \mathbf{G}$ and $\mathbf{D}\mathbf{u}_{opt}$ respectively. Note that the cardinality of F and T are $N_c^e - 1$ and $N_r^e - 1$, respectively. The instantaneous SINR of the cognitive link given by

$$\text{SINR}_c = \frac{P_c t^* H f^* H^* t}{t^* t} \quad (12)$$

And it can be maximized by performing an exhaustive search in F and T . Both the secondary beamforming vectors should be designed with interference signal as nullified condition. Beam forming vectors are selected to increase the maximum sum rate of the entire system.

$$\{f_{discrete}, t_{discrete}\} = \underset{f \in F, t \in T}{\operatorname{argmax}} \left\{ \frac{P_c t^* H f^* H^* t}{t^* t} \right\} \quad (13)$$

Note that for $N_c^e = N_r^e = 2$, there is only one vector in the set F and T . In general, $(N_c^e - 1) \times (N_r^e - 1)$ Computations are required to obtain the best beamformers $\mathbf{f}_{discrete}$ and $\mathbf{t}_{discrete}$. Although zero interference can always be guaranteed at both receivers by selecting the beamformer pair's \mathbf{f} , \mathbf{t} as in the above equation, the obtained solution is not optimal in the sense of maximum sum rate because the search in above is not carried out over the entire null space.

3.4 Gradient Algorithm

Since any vector in the null space of $\mathbf{V}_{opt}^* \mathbf{G}$ and $\mathbf{D}\mathbf{u}_{opt}$ satisfies the zero interference condition, there could be potentially other vectors in those spaces which yield a higher SINRC than $\mathbf{f}_{discrete}$ and $\mathbf{t}_{discrete}$. Suppose the columns of $\tilde{\mathbf{G}}$ and $\tilde{\mathbf{D}}$ contain the basis vectors of the null

space of $\mathbf{v}_{opt}^* \mathbf{G}$ and \mathbf{D} uopt, respectively. The optimal beamformers are in the form of

And for t is given by

$$t_{grad} = \frac{\bar{D}b}{\sqrt{b^*b}} \tag{15}$$

Where $\mathbf{a} \in \mathbb{C}^{(N_f-1) \times 1}$ and $\mathbf{b} \in \mathbb{C}^{(N_f-1) \times 1}$. The constrained optimization problem in the above equation can now be formulated as an unconstrained one whose goal is to find $\mathbf{a} \in \mathbb{C}^{(N_f-1) \times 1}$ and $\mathbf{b} \in \mathbb{C}^{(N_f-1) \times 1}$ such that the objective function in the above equation is maximized.

The equations is given by

$$\{\mathbf{a}_{opt}, \mathbf{b}_{opt}\} = \underset{\mathbf{a}, \mathbf{b}}{\operatorname{argmax}} \left\{ \frac{p_e \mathbf{b}^* \bar{D}^* \mathbf{H} \bar{G} \mathbf{a} \mathbf{a}^* \bar{G}^* \mathbf{H}^* \bar{D} \mathbf{b}}{\mathbf{b}^* \mathbf{b} \mathbf{a}^* \mathbf{a} \sigma_s^2} \right\}$$

The gradient algorithm is given by

$$\begin{bmatrix} \mathbf{a}[i+1] \\ \mathbf{b}[i+1] \end{bmatrix} = \begin{bmatrix} \mathbf{a}[i] \\ \mathbf{b}[i] \end{bmatrix} + \mu \begin{bmatrix} \frac{\partial f(\mathbf{a}[i], \mathbf{b}[i])}{\partial \mathbf{a}[i]^*} \\ \frac{\partial f(\mathbf{a}[i], \mathbf{b}[i])}{\partial \mathbf{b}[i]^*} \end{bmatrix} \tag{16}$$

In the equation 'i' is the iteration index and μ is the adaptation step size. Furthermore the two gradients in the above equation can be rewrite as

$$\frac{\partial f(\mathbf{a}[i], \mathbf{b}[i])}{\partial \mathbf{a}[i]^*} = \kappa \left[((\mathbf{b}^* \mathbf{b} \mathbf{a}^*) (\bar{G}^* \mathbf{H}^* \bar{D} \mathbf{b} \mathbf{b}^* \bar{D}^* \mathbf{H} \bar{G} \mathbf{a}) - (\mathbf{a}^* \bar{G}^* \mathbf{H}^* \bar{D} \mathbf{b} \mathbf{b}^* \bar{D}^* \mathbf{H} \bar{G} \mathbf{a}) (\mathbf{b}^* \mathbf{b} \mathbf{a}^*) \right] \tag{17}$$

$$\frac{\partial f(\mathbf{a}[i], \mathbf{b}[i])}{\partial \mathbf{b}[i]^*} = \kappa \left[(\mathbf{b}^* \mathbf{b} \mathbf{a}^*) (\bar{D}^* \mathbf{H}^* \bar{G} \mathbf{a} \mathbf{a}^* \bar{G}^* \mathbf{H}^* \bar{D} \mathbf{b}) - (\mathbf{b}^* \bar{D}^* \mathbf{H}^* \bar{G} \mathbf{a} \mathbf{a}^* \bar{G}^* \mathbf{H}^* \bar{D} \mathbf{b}) (\mathbf{a}^* \mathbf{b} \mathbf{a}^*) \right] \tag{18}$$

Furthermore, the two gradients are explained in the previous section can be explained in above, from which 'K' is an irrelevant constant. The time index i is dropped in the two gradients for case of presentation. In the next section some guidelines in choosing and adaptation constant μ and the initial values a [1] and b [1] are provided.

3.5 Simulation Result

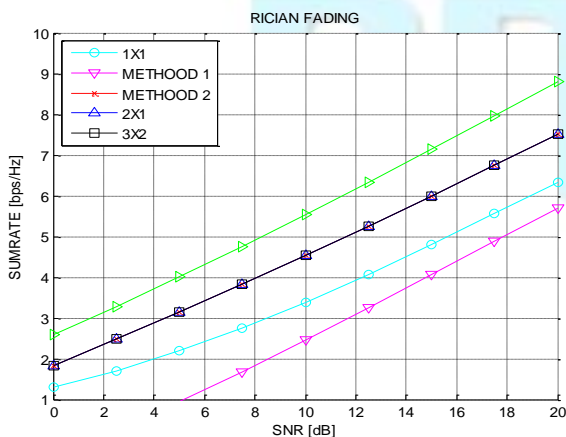


Fig.4.Sumrate for Various Methods Of Beamforming (Rician Fading Channel)

4. Conclusions

In this literature, we considered interference cancellation and achievable rate maximization via uncoordinated beam forming in a cognitive network which consists of a primary and secondary user. The secondary (cognitive) user was allowed to transmit concurrently with the primary licensed user. The beam forming vectors of the cognitive user were designed such that the interference is completely nullified both at the primary and secondary receivers while maximizing the rate of the cognitive link. Since no interference is created at the primary receiver, traditional approaches can be used to design the beam forming vectors or pre-coding matrices of the primary user. Three approaches were proposed for the design of the beam forming vectors of the cognitive link. Finally, it is noted that we motivate the uncoordinated beam forming and rate maximization concept in a cognitive network. However, the results can also be applied to practical systems, e.g., small cell deployment in a macro network.

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